An analytic technique to separate cochannel FM signals

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Abstract

A new technique is presented that can perfectly separate two **cochannel FM signals** in a noiseless environment. The method involves the **analytic solution** of the phases to within one of two possibilities, and a two-state **trellis algorithm** to trace the correct sequence of phase solutions. This method represents an improvement over other cochannel interference **separation** methods (e.g., joint Viterbi and cross-coupled phase-locked loop) which can never perfectly separate the signals. Simulations confirm the separation capability of the analytic technique.

1 Introduction

The complex baseband representation of a sampled cochannel FM signal is

$$r[n] = A[n]e^{j\theta[n]} + B[n]e^{j\phi[n]}. \tag{1}$$

When no confusion can result, the subscript n will be dropped. Initially, we shall assume that there is no noise and that A and B are known and vary slowly compared to θ and ϕ. Given A, B and r, accurate estimates of θ and ϕ are desired.

2 Analysis

One might expect, and indeed it turns out that, with one equation (Equation (1)) and two unknowns (θ and ϕ), there are two possible solutions for the phases. However, previous work

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[1,2,5,6] has not attempted to explicitly determine these solutions. In this section we show how to derive the phase solutions directly and exactly, without use of trellises or tracking loops. From Equation (1), we have

$$
\begin{aligned}
\|r\|^2 &= \|Ae^{j\theta} + Be^{j\phi}\|^2 \\
&= A^2 + B^2 + 2(Ae^{j\theta}) \cdot (Be^{j\phi}) \\
&= A^2 + B^2 + 2AB\cos(\phi - \theta),
\end{aligned}
$$

and thus

$$
\cos(\phi - \theta) = \frac{\|r\|^2 - A^2 - B^2}{2AB}. \tag{2}
$$

From $r = e^{j\theta}(A + Be^{j(\phi-\theta)})$, we see that

$$
\begin{aligned}
e^{j\theta} &= \frac{r}{A + Be^{j(\phi-\theta)}} \\
&= \frac{r}{A + Be^{j(\phi-\theta)}}\left(\frac{A + Be^{-j(\phi-\theta)}}{A + Be^{-j(\phi-\theta)}}\right) \\
&= \frac{r(A + Be^{-j(\phi-\theta)})}{A^2 + B^2 + 2AB\cos(\phi - \theta)} \\
&= \frac{r(A + Be^{-j(\phi-\theta)})}{\|r\|^2}.
\end{aligned}
$$

Hence,

$$
\theta = \arg[r(A + Be^{-j(\phi-\theta)})] = \arg[r(A + B\cos(\phi - \theta) - jB\sin(\phi - \theta))].
$$

By the symmetry of Equation (1), we may also immediately write:

$$
\phi = \arg[r(B + A\cos(\phi - \theta) + jA\sin(\phi - \theta))].
$$

By Letting $D = \cos(\phi - \theta) = (\|r\|^2 - A^2 - B^2)/(2AB)$, from Equation (2) we have

$$
\theta = \arg\left[r(A + BD \pm jB\sqrt{1 - D^2})\right] \tag{3}
$$

$$
\phi = \arg\left[r(B + AD \mp jA\sqrt{1 - D^2})\right]. \tag{4}
$$

We have thus determined the phases exactly, to within one of two possibilities.

3 The Tracking Algorithm

For a single sample $r[n]$, there is no reason to prefer one solution for $\theta[n]$ and $\phi[n]$ over the other possible solution. However, a sequence of solutions $\theta[n-2]$, $\theta[n-1]$, $\theta[n]$, ..., and $\phi[n-2]$, $\phi[n-1]$, $\phi[n]$, ..., can be chosen that has the bandwidth (or spectral density, if known) we expect for the underlying modulated phase.

We set up a two-state trellis, in which the first state represents the solution

$$\theta = \arg\left[r(A + BD + jB\sqrt{1 - D^2})\right]$$
$$\phi = \arg\left[r(B + AD - jA\sqrt{1 - D^2})\right].$$

and the second state represents the solution

$$\theta = \arg\left[r(A + BD - jB\sqrt{1 - D^2})\right]$$
$$\phi = \arg\left[r(B + AD + jA\sqrt{1 - D^2})\right].$$

The sequence of solution choices will be traced through the trellis using a Viterbi algorithm.

We want to choose the solution at time n that minimizes the disagreement between the instantaneous frequency hypothesized by the solution and the instantaneous frequency predicted from previous tentatively chosen values of the instantaneous frequency. Thus, at each state at time n, we store the phase solutions $(\hat{\theta}[n], \hat{\phi}[n])$ as determined by Equations (3) and (4), as well as the instantaneous frequencies $(\hat{\theta}'[n], \hat{\phi}'[n])$ and the predicted instantaneous frequencies $(\hat{\theta}'[n+1], \hat{\phi}'[n+1])$. The instantaneous frequency at time n is approximated from the phase using finite differences, and the predicted instantaneous frequency is calculated using an mth order Levinson-Durbin linear predictive coder (LPC), which has the form

$$\hat{\theta}'[n+1] = \sum_{i=1}^{m} a_i \hat{\theta}'[n+1-i].$$

A similar LPC is used for $\hat{\phi}'[n+1]$.

The Levinson-Durbin LPC is the linear minimum-mean squared error (LMMSE) estimator for the phases. The coefficients a_i are determined with a standard technique, as follows. Let $r_i = E(\theta'[n]\theta'[n-i])$, let $v = (r_1, \ldots, r_m)^T$, and let

$$R_m = \begin{bmatrix} r_0 & r_1 & \cdots & r_{m-1} \\ r_1 & r_0 & \cdots & r_{m-2} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m-1} & r_{m-2} & \cdots & r_0 \end{bmatrix}.$$

Then the coefficients are given by $a = R_{k-1}^{-1}v$. There is an efficient iterative technique to determine the mth order coefficients from the $(m-1)$th order coefficients, so that computing the inverse of the large autocorrelation matrix is not necessary [3]. To estimate r_i we assume a flat power spectrum density for θ' with a sharp cutoff at B Hz. Taking the inverse Fourier transform gives $r_i = \text{sinc}(2iBT_s)$, where T_s is the sampling rate. For example, if the bandwidth is 4 kHz. and $T_s = 1/132300$ sec., then for a fourth order LPC we have $v = (0.993996, 0.976115, 0.946741, 0.906506)^T$ and $a = (3.96459, -5.92481, 3.95553, -0.995421)^T$. If more is known about the spectral characteristics of θ', then more accurate coefficients may be determined.

Table 1: Performance results for five variations of the Ambassador-Brit test case.

Sub-case	Frequency Deviation (kHz)	SIR (dB)	Joint Viterbi (MSE)	Analytical Technique (MSE)
A	12	6	0.09/0.45	0.00/0.00
B	12	1	0.59/0.66	0.00/0.00
C	8	6	0.16/0.96	0.00/0.00
D	8	1	0.71/0.97	0.00/0.00
E	12	30	$-/-$	0.00/0.00

The branch metric from state i at time $n-1$ to state j and time n is the squared difference between the Levinson-Durbin prediction for $\theta'[n]$ from state i and the hypothesized solution $\theta'[n]$ from state j, added to the similar squared difference for $\phi'[n]$. The Viterbi algorithm operates by computing the four branch metrics at each time step, storing an accumulated metric, and tracing backwards in the trellis to find the correct phase solution.

4   Simulations and Conclusions

This analytic method has been coded in C and simulations have been run on a Pentium 166 machine. Parameters that can be passed to the program include the sampling rate, decoding delay, and order of the linear prediction; and the amplitudes and modulating signal bandwidths of the cochannel signals. The program reads in all parameters, computes the Levinson-Durbin LPC coefficients from the sampling rate and bandwidths, and then begins the Viterbi algorithm as described in the previous section.

There were five test cases for FM voice signals. In each case the the first voice is the Iraqi ambassador and the second voice is a British woman. In all cases, the sampling rate is 132300 Hz., (to match with previous tests for the joint Viterbi and CCPLL), the SNR is infinity, a fifth order LPC is used under the assumption of flat 4 kHz. bandwidth modulating signals, and the decoding delay is 1. The SIR and frequency deviations were varied. Table 1 gives the normalized mean-squared error (MSE) between the true and estimated instantaneous frequencies of the dominant and subdominant signals, and compares it to the joint Viterbi algorithm [4]. In all sub-cases, both the dominant and subdominant signals were separated perfectly, to within the floating point precision of the computer (normalized MSE of 10^{-20} or less), i.e., the correct branch of the trellis was chosen at every step. In addition, the one-step linear predictor itself is very accurate; in every subcase, the average difference between the linearly predicted phase and the phase given by the chosen state is 1.5 degrees or less.

This new analytic technique is capable of perfect separation in a noiseless environment. Additional research and simulation is needed, however, to provide for estimation of unknown amplitudes. One simple technique to estimate the amplitudes is to find the maximum and minimum of $\|r\|$ over a large set of samples. For very low noise, the minimum value is approximately $A - B$

and the maximum value is approximately $A + B$. The values of A and B can then be estimated by adding and subtracting these numbers. Or amplitude estimation can be included within the trellis itself, using a gradient descent algorithm.

Additional work is also needed to determine the resistance of the algorithm to the introduction of noise. Clearly, noise can cause inaccuracies in the analytic solutions; however, there seems promise for improvements over other techniques.

References

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